

Graphing $f(x) = |x|$ and Key Attributes**TEACHER KEY** | Algebra II Honors | Unit 2 | Day 1 | TEK A2.2.A

Objective: I can analyze the two pieces of $f(x) = |x|$ separately - their restricted domains and ranges - combine them into the domain and range of $f(x)$, and test whether $f(x)$ is even, odd, or neither.

Vocabulary

Piecewise function: a function defined by **different rules** on different parts of the domain.

Restricted domain: the domain a single piece applies to, e.g. **$x \geq 0$** for one piece of $|x|$.

Even function: $f(-x) = f(x)$ for every x - symmetric about the y-axis.

Odd function: $f(-x) = -f(x)$ for every x - symmetric about the origin.

KEY IDEA

$f(x) = |x|$ has two pieces: $y = x$ restricted to $x \geq 0$, and $y = -x$ restricted to $x < 0$.

Each piece is its OWN linear function with its OWN domain and range - analyze them separately, then combine.

Test even/odd with $f(-x)$: $f(-x) = f(x) \rightarrow$ EVEN. $f(-x) = -f(x) \rightarrow$ ODD. Neither $\rightarrow$ neither.

Let's Work Through It

Example 1 — Analyze each piece of $f(x) = |x|$ separately: its restricted domain, and the range IT alone produces.

Piece $y = x$: restricted domain **$x \geq 0$** its range: **$y \geq 0$**

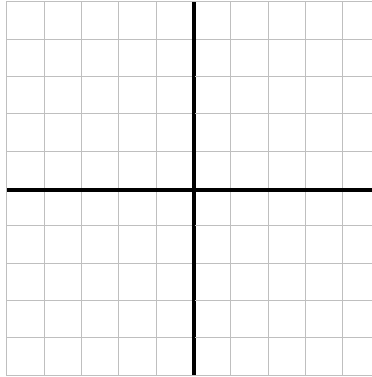
Piece $y = -x$: restricted domain **$x < 0$** its range: **$y > 0$ (x is negative, so $-x$ is positive)**

Example 2 — Combine the two pieces' restricted domains and ranges into the domain and range of $f(x) = |x|$ as a whole.

Domain: $x \geq 0$ combined with $x < 0 \rightarrow$ **$(-\infty, \infty)$**

Range: $y \geq 0$ combined with $y > 0 \rightarrow$ **$[0, \infty)$**

Example 3 — Graph ONLY $y = x$ on its restricted domain $x \geq 0$, and ONLY $y = -x$ on its restricted domain $x < 0$. Each square is 1 unit; both axes run -5 to 5.



Vertex: **(0, 0)** Only intercept: **(0, 0)** , since $f(0) = 0$ in either piece

Example 4 — Test whether $f(x) = |x|$ is even, odd, or neither.

$f(-x) = |-x| = \mathbf{|x|}$ So $f(-x) = \mathbf{f(x)}$ (write $f(x)$ or $-f(x)$)

Conclusion: **even** Symmetric about the: **y-axis**

Example 5 — Case analysis: for which values of x does $|x| = x$ (the two pieces agree)?

$x \geq 0$ - the two pieces give the same output only where x is already non-negative

Your Turn

You Try 1 — Case analysis: for which values of x does $|x| = -x$?

$x \leq 0$

You Try 2 — Explain why $f(x) = |x|$ has a minimum value, but a line $y = mx + b$ ($m \neq 0$) does not.
