

Transformations of $f(x) = |x|$, Part II

TEACHER KEY | Algebra II Honors | Unit 2 | Day 3 | TEK A2.6.C

Objective: I can read the vertex and the transformations off vertex form, and work backward from a vertex and one point to write the equation.

Vocabulary

Vertex form: $f(x) = a|x - c| + d$, written so the vertex **(c, d)** reads off directly.

Horizontal shift: c moves the graph **opposite** the sign you see: $x - 5$ means **right 5**.

Vertical shift: d moves the graph **the same as** the sign you see: -7 means **down 7**.

Working backward: using a vertex and one other point to solve for the missing **a**.

KEY IDEA

Vertex form in action:

$f(x) = 3|x - 2| + 1 \rightarrow$ vertex (2, 1); stretch by 3, right 2, up 1.

$f(x) = -0.5|x + 4| - 3 \rightarrow$ vertex (-4, -3); compress by 0.5, flip over the x-axis, left 4, down 3.

Working backward - 4 steps:

Step 1 - Find the vertex (that is c and d).

Step 2 - Plug the other point's x and y into $a|x - c| + d$.

Step 3 - Solve for a.

Step 4 - Rewrite $a|x - c| + d$ with your a, c, and d.

Let's Work Through It

Example 1 — Given $f(x) = 4|x - 3| - 2$, write the vertex and list every transformation from the parent.

Vertex: **(3, -2)** Opens: **up ($a > 0$)**

Transformations: **vertical stretch by 4, shifted right 3, shifted down 2**

Example 2 — Given $f(x) = -(1/2)|x + 5| + 6$, write the vertex and list every transformation from the parent.

Vertex: **(-5, 6)** Opens: **down ($a < 0$)**

Transformations: **vertical compression by 1/2, reflected over the x-axis, shifted left 5, shifted up 6**

Example 3 — A function has vertex (3, -2) and passes through (5, 4). Write its equation, using all four steps.

Step 1 - Find the vertex: $c = 3$, $d = -2$

Step 2 - Plug in x and y: $a|5 - 3| + (-2) = 4$

Step 3 - Solve for a: $2a - 2 = 4 \rightarrow a = 3$

Step 4 - Rewrite: $f(x) = 3|x - 3| - 2$

Example 4 — A function has vertex (-1, 5) and passes through (2, -4). Write its equation, using all four steps.

Step 1 - Find the vertex: $c = -1$, $d = 5$

Step 2 - Plug in x and y: $a|2 - (-1)| + 5 = -4$

Step 3 - Solve for a: $3a + 5 = -4 \rightarrow a = -3$

Step 4 - Rewrite: $f(x) = -3|x + 1| + 5$

Your Turn

You Try 1 — Given $f(x) = -1.2|x - 3| + 8$, write the vertex and list every transformation from the parent.

Vertex: **(3, 8)**

Transformations: **vertical stretch by 1.2, reflected over the x-axis, shifted right 3, shifted up 8**

You Try 2 — A function has vertex (4, 1) and passes through (7, 10). Write its equation, using all four steps.

Step 1: $c = 4$, $d = 1$ Step 2: $a|7 - 4| + 1 = 10$

Step 3: $3a + 1 = 10 \rightarrow a = 3$ Step 4: $f(x) = 3|x - 4| + 1$