

Extraneous Solutions

TEACHER KEY | Algebra II Honors | Unit 2 | Day 6 | TEK A2.6.D, A2.6.E

Objective: I can solve absolute value equations with a variable on both sides, identify which candidates are extraneous, and confirm the real solutions in Desmos.

Vocabulary

Extraneous solution: a value the algebra produces that does **NOT** satisfy the **original** equation.

Candidate: an answer a branch gives, before it has been **checked** - it is not a solution yet.

Empty set: the solution set when **every** candidate turns out extraneous.

KEY IDEA

Step 1 - Isolate: get the absolute value alone on one side.

Step 2 - Split: $|A| = B$ becomes $A = B$ OR $A = -B$.

Step 3 - Solve: solve each branch for x. These are candidates, not answers yet.

Step 4 - Check: plug EVERY candidate into the ORIGINAL equation. Reject any that fail.

Step 5 - Desmos check: y_1 = left side, y_2 = right side. A real solution is a crossing.

Let's Work Through It

Example 1 — Review from Day 5: solve $|3x - 10| = 2x - 5$.

Step 2 - Split: $3x - 10 = 2x - 5$ OR $3x - 10 = -(2x - 5)$

Step 3 - Solve: $x = 5$ or $5x = 15 \rightarrow x = 3$

Step 4 - Check: $x = 5$: $|5| = 5$, $2(5) - 5 = 5 \checkmark$ $x = 3$: $|-1| = 1$, $2(3) - 5 = 1 \checkmark$

Example 2 — Solve $|x + 2| = 2x + 1$.

Step 2 - Split: $x + 2 = 2x + 1$ OR $x + 2 = -(2x + 1)$

Step 3 - Solve: $x = 1$ or $3x = -3 \rightarrow x = -1$

Step 5 - Desmos: $y_1 = |x + 2|$, $y_2 = 2x + 1$ cross only at $x = 1$ Solution set: $\{1\}$

Example 3 — Solve $|x + 6| = 2x + 3$.

Step 2 - Split: $x + 6 = 2x + 3$ OR $x + 6 = -(2x + 3)$

Step 3 - Solve: $x = 3$ or $x = -3$

Step 4 - Check: $x = 3$: $|9| = 9$, $2(3) + 3 = 9 \rightarrow$ **valid**

$x = -3$: $|3| = 3$, $2(-3) + 3 = -3 \rightarrow$ **EXTRANEIOUS**

Example 4 — Solve $|x - 1| = 2x + 4$. Watch which branch fails this time.

Step 2 - Split: $x - 1 = 2x + 4$ OR $x - 1 = -(2x + 4)$

Step 3 - Solve: $x = -5$ or $3x = -3 \rightarrow x = -1$

Step 5 - Desmos: $y_1 = |x - 1|$, $y_2 = 2x + 4$ cross only at $x = -1$ Solution set: $\{-1\}$

Your Turn

You Try 1 — Solve $|2x - 1| = x + 4$, checking every candidate and confirming in Desmos.

Candidates: $x = 5$ and $x = -1$ After checking: **both are valid**

Desmos: $y_1 = |2x - 1|$, $y_2 = x + 4$ cross **twice** Solution set: $\{-1, 5\}$

You Try 2 — Solve $|x - 6| = -x - 2$. Does any candidate survive the check?

Split: $x - 6 = -x - 2 \rightarrow x = 2$ OR $x - 6 = x + 2 \rightarrow$ **no candidate ($-6 \neq 2$)**

Check $x = 2$: $|-4| = 4$, but $-2 - 2 = -4$, so it is **EXTRANEIOUS**

Solution set: **empty set** Desmos: **y_1 and y_2 never cross**