

Name _____ Date _____ Period _____

Extraneous Solutions

Algebra II Honors | Unit 2 | Day 6 | TEK A2.6.D, A2.6.E

Objective: I can solve absolute value equations with a variable on both sides, identify which candidates are extraneous, and confirm the real solutions in Desmos.

Vocabulary

Extraneous solution: a value the algebra produces that does _____ satisfy the _____ equation.

Candidate: an answer a branch gives, before it has been _____ - it is not a solution yet.

Empty set: the solution set when _____ candidate turns out extraneous.

KEY IDEA

Step 1 - Isolate: get the absolute value alone on one side.

Step 2 - Split: $|A| = B$ becomes $A = B$ OR $A = -B$.

Step 3 - Solve: solve each branch for x. These are candidates, not answers yet.

Step 4 - Check: plug EVERY candidate into the ORIGINAL equation. Reject any that fail.

Step 5 - Desmos check: y_1 = left side, y_2 = right side. A real solution is a crossing.

Let's Work Through It

Example 1 — Review from Day 5: solve $|3x - 10| = 2x - 5$.

Step 2 - Split: $3x - 10 = 2x - 5$ OR $3x - 10 =$ _____

Step 3 - Solve: $x =$ _____ or $5x = 15 \rightarrow x =$ _____

Step 4 - Check: $x = 5$: $|5| = 5$, $2(5) - 5 = 5 \checkmark$ $x = 3$: $|-1| = 1$, $2(3) - 5 = 1 \checkmark$

Example 2 — Solve $|x + 2| = 2x + 1$.

Step 2 - Split: $x + 2 = 2x + 1$ OR $x + 2 =$ _____

Step 3 - Solve: $x =$ _____ or $3x = -3 \rightarrow x =$ _____

Step 5 - Desmos: $y_1 = |x + 2|$, $y_2 = 2x + 1$ cross only at $x =$ _____ Solution set: { _____ }

Example 3 — Solve $|x + 6| = 2x + 3$.

Step 2 - Split: $x + 6 = 2x + 3$ OR $x + 6 =$ _____

Step 3 - Solve: $x =$ _____ or $x =$ _____

Step 4 - Check: $x = 3$: $|9| = 9$, $2(3) + 3 = 9 \rightarrow$ _____

$x = -3$: $|3| = 3$, $2(-3) + 3 = -3 \rightarrow$ _____

Example 4 — Solve $|x - 1| = 2x + 4$. Watch which branch fails this time.

Step 2 - Split: $x - 1 = 2x + 4$ OR $x - 1 =$ _____

Step 3 - Solve: $x =$ _____ or $3x = -3 \rightarrow x =$ _____

Step 5 - Desmos: $y_1 = |x - 1|$, $y_2 = 2x + 4$ cross only at $x =$ _____ Solution set: {_____}

Your Turn

You Try 1 — Solve $|2x - 1| = x + 4$, checking every candidate and confirming in Desmos.

Candidates: $x =$ _____ and $x =$ _____ After checking: _____

Desmos: $y_1 = |2x - 1|$, $y_2 = x + 4$ cross _____ Solution set: {_____}

You Try 2 — Solve $|x - 6| = -x - 2$. Does any candidate survive the check?

Split: $x - 6 = -x - 2 \rightarrow x =$ _____ OR $x - 6 =$ _____ $\rightarrow$

Check $x = 2$: _____

Solution set: _____ Desmos: _____