

Name _____ Date _____ Period _____

Solving Absolute Value Linear Inequalities

TEACHER KEY | Algebra II Honors | Unit 2 | Day 7 | TEK A2.6.F

Objective: I can solve an absolute value linear inequality with either the sandwich method or the split method, and graph the solution on a number line.

Vocabulary

"And" statement: true only if BOTH parts are true - used for $<$ or $\leq$. Since $|x|$ is the distance from x to 0, $|x| < k$ means x is CLOSER to 0 than k , so it sits between $-k$ and k .

"Or" statement: true if EITHER part is true - used for $>$ or $\geq$. $|x| > k$ means x is FARTHER from 0 than k , so it is past k or past $-k$.

Sandwich method: write an "and" as ONE inequality: $-k < A < k$.

Split method: write the two branches separately, then combine them.

KEY IDEA

Step 1 - Isolate: get the absolute value alone.

Step 2 - Split: for $<$ or $\leq$ use AND - either as one sandwiched inequality, or as two branches. For $>$ or $\geq$ use OR - two separate branches.

Step 3 - Solve: solve each part, flipping the symbol any time you multiply or divide by a negative.

Step 4 - Graph: mark the solution on a number line, then write the final answer.

Let's Work Through It

Example 1 — Solve $2|x + 1| - 3 < 7$ two ways: the sandwich method.

Step 1 - Isolate: $|x + 1| < 5$

Sandwich: $-5 < x + 1 < 5 \rightarrow -6 < x < 4$

-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
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Example 2 — Solve $5|2x - 3| + 2 \leq 27$ two ways: the split method.

Step 1 - Isolate: $5|2x - 3| \leq 25 \rightarrow |2x - 3| \leq 5$

Split: $2x - 3 \leq 5$ AND $-(2x - 3) \leq 5$

Solve each (flip when you divide by the negative): $x \leq 4$ AND $x \geq -1$

Combine: $-1 \leq x \leq 4$ - same answer either way

-3	-2	-1	0	1	2	3	4	5	6
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Example 3 — Solve $4|2x - 5| - 6 > 14$.

Step 1 - Isolate: $4|2x - 5| > 20 \rightarrow |2x - 5| > 5$

Step 2 - Split (OR): $2x - 5 > 5$ OR $2x - 5 < -5$

Step 3 - Solve: $x > 5$ or $x < 0$

Example 4 — Solve $3|4 - x| + 2 \geq 17$. The inside is written $4 - x$, not $x - 4$, so watch the flip.

Step 1 - Isolate: $3|4 - x| \geq 15 \rightarrow |4 - x| \geq 5$

Step 2 - Split (OR): $4 - x \geq 5$ OR $4 - x \leq -5$

Step 3 - Solve (flip both times you divide by -1): $x \leq -1$ or $x \geq 9$

Your Turn

You Try 1 — Solve $2|x - 4| + 1 \leq 9$, using whichever method you prefer.

Isolate: $|x - 4| \leq 4$ Solution: $0 \leq x \leq 8$

You Try 2 — Using the distance meaning of absolute value, explain why $<$ gives an AND but $>$ gives an OR.
