Name _____ Date _____ Period ____

Absolute Value Inequalities: Special Cases and Interval Notation

TEACHER KEY | Algebra II Honors | Unit 2 | Day 8 | TEK A2.6.F

Objective: I can decide whether an absolute value inequality has no solution or all real numbers, and write ordinary solutions in interval notation.

Vocabulary

No solution: NO value of x works at all. Written as the **empty set**.

All real numbers: EVERY value of x works. Written as **$(-\infty, \infty)$**.

KEY IDEA

Step 1 - Isolate: get the absolute value completely alone. If you divide by a negative, flip the symbol.

Step 2 - Look at the number on the other side. If it is POSITIVE, this is an ordinary Day 7 problem - solve it normally.

Step 3 - If that number is NEGATIVE, read the symbol instead of solving:

|expression| < or ≤ a negative → NO SOLUTION. An absolute value is never negative.

|expression| > or ≥ a negative → ALL REAL NUMBERS. An absolute value is always at least 0.

Let's Work Through It

Example 1 — Solve $|2x - 3| < 3$ and write the answer in interval notation.

Step 1 - Isolate: the absolute value is already alone.

Sandwich (AND, since <): $-3 < 2x - 3 < \mathbf{3}$

Solve: $0 < 2x < \mathbf{6}$ → **$0 < x < 3$**

Interval notation: **$(0, 3)$**

-2	-1	0	1	2	3	4	5

Example 2 — Solve $8 - 2|3x - 1| \leq 10$.

Step 1 - Isolate: $-2|3x - 1| \leq \mathbf{2}$ → divide by -2 and FLIP → $|3x - 1| \geq \mathbf{-1}$

Step 2 - The number on the other side is **negative, so read the symbol instead of solving**

Step 3 - Read the symbol: **≥ a negative, and an absolute value is always at least 0, so ALL REAL NUMBERS**

Interval notation: **$(-\infty, \infty)$**

Example 3 — Solve $3|2x + 5| + 10 \leq 4$.

Step 1 - Isolate: $3|2x + 5| \leq \mathbf{-6}$ → $|2x + 5| \leq \mathbf{-2}$

Step 2 - The number on the other side is **negative, so read the symbol instead of solving**

Step 3 - Read the symbol: **≤ a negative, and an absolute value is never negative, so NO SOLUTION**

Your Turn

You Try 1 — Solve $|x - 2| + 6 \leq 1$.

Isolate: $|x - 2| \leq \mathbf{-5}$ Solution: **no solution**

You Try 2 — Solve $|x - 2| + 6 \geq 1$. Same left side as You Try 1, just a flipped symbol.

Isolate: $|x - 2| \geq \mathbf{-5}$ Solution: **all real numbers** Interval: **$(-\infty, \infty)$**