Name ________________________________ Date _________ Period ____

Extraneous Solutions and Real-World Applications

TEACHER KEY | Algebra II | Unit 2 | Day 6 | TEK A2.6.D, A2.6.E

Objective: I can identify extraneous solutions when solving absolute value equations, and solve the real-world equations from Day 4.

Vocabulary

Extraneous solution: a value the algebra produces that does **NOT** actually satisfy the **original** equation.

KEY IDEA

Step 1 - Split: $|A| = B$ becomes $A = B$ OR $A = -B$, same as always.

Step 2 - Solve: solve each branch for x.

Step 3 - Check: plug EVERY answer into the ORIGINAL equation - reject any that fail.

Step 4 - Desmos check: graph each side as y1/y2 - real solutions are where they cross.

Let's Work Through It

Example 1 — Solve $3|x - 1| - 2 = 6x + 1$. The variable is on both sides, so check every answer.

Isolate: $3|x - 1| = \mathbf{6x + 3}$ → $|x - 1| = \mathbf{2x + 1}$

Split: $x - 1 = 2x + 1$ OR $x - 1 = \mathbf{-(2x + 1)}$

Solve: $x = \mathbf{-2}$ or $x = \mathbf{0}$

Check $x = -2$: $3|-3| - 2 = 7$, $6(-2) + 1 = -11$. $7 \neq -11$ → **EXTRANEOUS**

Check $x = 0$: $3|-1| - 2 = 1$, $6(0) + 1 = 1$. $1 = 1$ → **valid** Solution set: $\{\mathbf{0}\}$

Desmos check: Type $y1 = 3|x - 1| - 2$ and $y2 = 6x + 1$ into two input boxes. The graphs cross only at $x = \mathbf{0}$ - there is no crossing at $x = -2$, confirming it was extraneous.

Example 2 — Recall Day 4: Clarissa wants a house within 5 houses of #1248 on an even-numbered street, giving $|x - 1248| = 5$. Solve, and check both against the situation.

$x = \mathbf{1253}$ or $x = \mathbf{1243}$ **both are even-numbered addresses within 5 houses, so both are valid**

Example 3 — Recall Day 4: a surveying instrument has a 1.5% margin of error on a 10-mile stretch, giving $1.5 = |x - 10|/10 \cdot 100$. Solve it.

$|x - 10| =$ **0.15** $x =$ **10.15** or $x =$ **9.85** miles

Example 4 — Recall Day 4: an AI demand-forecasting model is trusted only within 25 units of an actual demand of 400 units, giving $|x - 400| = 25$. Solve for the acceptable prediction range.

$x =$ **425** or $x =$ **375**   **both are reasonable demand values, so both are valid**

Example 5 — Solve $|x + 3| = 2x + 1$, and check both solutions.

Split: $x + 3 = 2x + 1$ OR $x + 3 =$ **$-(2x + 1)$**

Solve: $x =$ **2**   or $x =$ **$-4/3$**

Check $x = 2$: $|2 + 3| = 5$, $2(2) + 1 = 5$. $5 = 5$ → **valid**

Check $x = -4/3$: $|-4/3 + 3| = 5/3$, $2(-4/3) + 1 = -5/3$. $5/3 \neq -5/3$ → **EXTRANEOUS**

Solution set: {**2**}

Your Turn

You Try 1 — Solve $|2x - 1| = x + 4$, and check both solutions.

$x =$ **5** or $x =$ **-1**   Solution set: {**-1, 5**} **(both valid this time)**

You Try 2 — Solve $|x - 4| = 3x$, checking both solutions in the ORIGINAL equation. State the solution set.

$x =$ **1** or $x =$ **-2 (extraneous)**   Solution set: {**1**}