Name ____________________________________ Date _________ Period ____

Absolute Value Inequalities: Special Cases and Interval Notation

TEACHER KEY | Algebra II | Unit 2 | Day 8 | TEK A2.6.F

Objective: I can write absolute value inequality solutions in interval notation, solve harder cases, and recognize when an inequality has no solution.

Vocabulary

No solution: NO value of x works. Happens when an isolated absolute value is set **<** or **≤** a negative number.

All real numbers: an isolated absolute value set **>** or **≥** a negative number - true no matter what x is.

KEY IDEA

Step 1 - Isolate: get the absolute value alone.

Step 2 - Split: use the AND/OR rule from Day 7 to write a compound inequality.

Step 3 - Solve: solve each part, then graph it on a number line.

Step 4 - Combine: write interval notation - ( ) for strict, [ ] for ≥/≤. Isolating to < or ≤ a NEGATIVE number means no solution; isolating to > or ≥ a NEGATIVE number means all real numbers.

Let's Work Through It

Example 1 — Solve $|3x + 1| - 4 \le 2x - 1$. The variable is on both sides, so isolate first - both bounds exist here, unlike Examples 2-5.

Step 1 - Isolate: $|3x + 1| \le$ **$2x + 3$**

Step 2 - Split (AND, since ≤): **$-(2x + 3)$** $\le 3x + 1 \le 2x + 3$

Step 3 - Solve the left half: $-(2x+3) \le 3x+1$ → **$x \ge -4/5$**

Step 3 - Solve the right half: $3x+1 \le 2x+3$ → **$x \le 2$**

Step 4 - Combine: **[-4/5, 2]**

Step 5 - Spot check x = 0 (inside the interval): $|3(0)+1| - 4 =$ **-3**, $2(0) - 1 =$ **-1**. $-3 \le -1$ ✓

-3	-2	-1	0	1	2	3	4

Example 2 — Solve $3|2x + 5| + 10 \le 4$.

Isolate: $3|2x + 5| \le$ **-6** → $|2x + 5| \le$ **-2**

An absolute value can never be ≤ a negative number. Solution: **no solution**

Example 3 — Solve $8 - 2|3x - 1| \ge 10$.

Isolate: $-2|3x - 1| \ge$ **2** → dividing by -2 flips it → $|3x - 1| \le$ **-1**

An absolute value can never be ≤ a negative number. Solution: **no solution**

Example 4 — Solve $3|2x + 5| + 10 \ge 4$. Same left side as Example 2, just a flipped symbol.

Isolate: $3|2x + 5| \ge$ **-6** → $|2x + 5| \ge$ **-2**

An absolute value is ALWAYS ≥ a negative number. Solution: **all real numbers**

Example 5 — Solve $8 - 2|3x - 1| \le 10$. Same left side as Example 3, just a flipped symbol.

Isolate: $-2|3x - 1| \le$ **2** → dividing by -2 flips it → $|3x - 1| \ge$ **-1**

An absolute value is ALWAYS ≥ a negative number. Solution: **all real numbers**

Your Turn

You Try 1 — Solve $|2x - 1| - 3 \le x + 2$.

Isolate: $|2x - 1| \le$ **x + 5**

Split (AND): $-(x + 5) \le 2x - 1 \le$ **x + 5**

Solve the left half: $x \ge$ **-4/3** Solve the right half: $x \le$ **6**

Combine: **[-4/3, 6]**

You Try 2 — Solve $5 + |x - 3| \le 2$.

Isolate: $|x - 3| \le$ **-3**

Solution: **no solution**