Name ___________________________________ Date _________ Period ____

Absolute Value Inequalities: Special Cases and Interval Notation

Algebra II | Unit 2 | Day 8 | TEK A2.6.F

Objective: I can write absolute value inequality solutions in interval notation, solve harder cases, and recognize when an inequality has no solution.

Vocabulary

No solution: NO value of x works. Happens when an isolated absolute value is set ____ or ______ a negative number.

All real numbers: an isolated absolute value set ____ or ______ a negative number - true no matter what x is.

KEY IDEA

Step 1 - Isolate: get the absolute value alone.

Step 2 - Split: use the AND/OR rule from Day 7 to write a compound inequality.

Step 3 - Solve: solve each part, then graph it on a number line.

Step 4 - Combine: write interval notation - ( ) for strict, [ ] for ≥/≤. Isolating to < or ≤ a NEGATIVE number means no solution; isolating to > or ≥ a NEGATIVE number means all real numbers.

Let's Work Through It

Example 1 — Solve $|3x + 1| - 4 \le 2x - 1$. The variable is on both sides, so isolate first - both bounds exist here, unlike Examples 2-5.

Step 1 - Isolate: $|3x + 1| \le$ ________________

Step 2 - Split (sandwich, since ≤): ___________________ $\le 3x + 1 \le 2x + 3$

Step 3 - Solve the left half: $-(2x+3) \le 3x+1$ → ___________________

Step 3 - Solve the right half: $3x+1 \le 2x+3$ → _______________

Step 4 - Combine: ________________

Step 5 - Spot check x = 0 (inside the interval): $|3(0)+1| - 4 =$ ________ , $2(0) - 1 =$ ________ . $-3 \le -1$ ✓

-3	-2	-1	0	1	2	3	4

Example 2 — Solve $3|2x + 5| + 10 \le 4$.

Isolate: $3|2x + 5| \le$ ________ → $|2x + 5| \le$ ________

An absolute value can never be ≤ a negative number. Solution: ____________________

Example 3 — Solve $8 - 2|3x - 1| \ge 10$.

Isolate: $-2|3x - 1| \ge$ ______ → dividing by -2 flips it → $|3x - 1| \le$ ________

An absolute value can never be ≤ a negative number. Solution: ____________________

Example 4 — Solve $3|2x + 5| + 10 \ge 4$. Same left side as Example 2, just a flipped symbol.

Isolate: $3|2x + 5| \ge$ ________ → $|2x + 5| \ge$ ________

An absolute value is ALWAYS ≥ a negative number. Solution: ________________________

Example 5 — Solve $8 - 2|3x - 1| \le 10$. Same left side as Example 3, just a flipped symbol.

Isolate: $-2|3x - 1| \le$ ______ → dividing by -2 flips it → $|3x - 1| \ge$ ________

An absolute value is ALWAYS ≥ a negative number. Solution: ________________________

Your Turn

You Try 1 — Solve $|2x - 1| - 3 \le x + 2$.

Isolate: $|2x - 1| \le$ ____________

Split (AND): $-(x + 5) \le 2x - 1 \le$ ____________

Solve the left half: $x \ge$ __________ Solve the right half: $x \le$ ______

Combine: ________________

You Try 2 — Solve $5 + |x - 3| \le 2$.

Isolate: $|x - 3| \le$ ________

Solution: ____________________